Somatic Portfolio Theory:
When Emotions lead to Economic Efficiency

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Abstract: This paper offers a theoretical generalization of the mean-variance theory (MVT) by integrating the 'expected returns/risk' rule with variables that measure emotions. We validate its accuracy using a psycho-physiological experiment with a sample of 645 individuals who were asked to take portfolio decisions in a laboratory setting. Results show that MVT frequently fails to describe investor behavior. We obtain evidence that individuals actually take efficient portfolio choices, but only when emotions are added to the equation. This paper shows that by merging theories of rational choice and evidence of emotions, the authentic human decision process can be described and predicted.

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1 Introduction

This paper investigates if human decision-making, in economics, follows an optimizing process when emotions are explicitly considered. We take a classical optimization problem, namely the Mean-Variance Theory (MVT) proposed by Harry Markowitz in 1952 (Markowitz (1952)), as a theoretical basis. The Markowitz model is mainly inspired by a normative approach, referring to 'the rule that the investor does (or should) consider' (Markowitz (1952) p. 77). MVT assumes that individuals are able to understand a mean-variance framework and that they coherently follow an optimization process, i.e. they take choices by reducing risks (variance) for any given return. Mean-variance efficient portfolios are the result of such optimizing choices.

Consequently, our research question is: do investors in real decision-making follow the MVT and behave according to what they 'should' do?

In order to investigate this issue, we utilize the Markowitz 'normative' model based on the assumption that individuals' portfolio choices are driven by the 'expected returns-variance of returns' rule. Then we theoretically introduce a 'generalized' model which integrates MVT with individual emotions. Here we consider emotions in the Lowenstein (2000) meaning, i.e. 'immediate emotions', also referred to as 'visceral factors' (Lowenstein (1996)) involved in the act of decision making. Their effect is not transient and is based on the network synchronization between central and peripherical systems[1].

[1]In their seminal work, Varela et al. (2001) uncover a synchronization process, that

Technological developments offer non-ambiguous measurements of latent heterogeneous emotions activated within human decision-making, at least in terms of individuals' emotional response to stimulus. This condition allows us to empirically investigate the accuracy of the 'generalized' model compared to the 'normative' one. In order to reduce the noise induced by concurrent drivers in the financial decision process, we replicate in a laboratory setting the portfolio decision process, using a portfolio of 4 generic assets, with random pay-offs. By observing 645 individuals, we collected data on portfolio choices, in terms of returns and concomitant emotional activation. Specifically, we measured the emotional activation as the Skin Conductance Response (SCR) shown by individuals after gains and losses, as the somatic component driven by the autonomous nervous system. From neurological research, we know that decision-making is a process that is influenced by body-marker signals that arise in bioregulatory processes; this influence can occur both consciously and unconsciously (Bechara and Damasio (2005)).

Our findings show that between 11 and 13 per cent of the individuals' portfolio choices can be considered efficient according to the Markowitz 'normative' model, with 79-80 per cent of them affected by a 'severe' sub-efficiency. On the contrary, more than 84-85 per cent of the individuals' portfolios are efficient according to the 'generalized' model, which includes both monetary pay-offs and emotional reactions, and only 9-10 per cent of

solves a problem named 'large-scale integration', and describes neural mechanisms that select and coordinate this distributed brain activity to produce a flow of adapted and unified cognitive moments.

them suffer from a 'severe' sub-efficiency.

These findings provide supporting evidence that: i) the Markowitz normative approach generally fails to describe 'what an investor practically does'; ii) the MVT conceptual framework properly describes efficient behaviors *only* when it incorporates visceral factors, i.e. within the 'generalized' model; iii) individuals *actually* follow an optimizing decision process and take efficient portfolio choices, but the degree of efficiency strongly increases when emotions are considered. It follows that the human optimization process is not limited to monetary gains and losses, but it is also guided by additional emotional gratification.

Our 'generalized' model is able to 'describe' real-world outcomes because it is close to authentic human decision-making. At the same time, its theoretical lay-out will allow further research developments, as a new 'Somatic Portfolio Theory', where emotions and the body's signals converge towards economic efficiency.

2 Emotions in Human Decision Making

One could claim that the Markowitz decision framework includes emotions because it implicitly assumes that human beings dislike high variance, as high variance tends to increase feelings of anxiety or fear. Moreover, people desire expected returns due to the sense of excitement involved. But these emotions are 'rational' and predictable behaviors. As a proof of that, the MVT decision framework could be transformed into an algorithm for a software, able to precisely apply the 'expected returns-variance of returns'. Conversely, 'immediate emotions' may bring human being to unpredictable behaviors, sometimes being attracted by risk, or disregarding returns.

When cognitive studies approached the issue of economic decision-making, a set of limitations of rationality was uncovered. We refer to 'behavioral studies' which experimentally observed human behaviors and revealed many cognitive biases in individual decision making (Kahneman and Tversky (1974, 1979); Subrahmanyam (2008)). The presence of cognitive biases has also been developed in portfolio theories. Shefrin and Statman (2000) suggest a 'Behavioral Portfolio Theory', a positive model in which they include the possibility that individuals build separated portfolios, in relation to different mental accounts, i.e. different investment goals. Their model, from a single mental account to a multiple mental account version, would allow to theoretically justify the Friedman-Savage puzzle (Friedman and Savage (1948)), the paradox of having investors holding insurance and buying lotteries at the same time. In 2010, Das, Markowitz, Scheid and Statman propose a unified mental account portfolio theory as a convergence of the Markowitz MVT and the Behavioral Portfolio Theory.

Moving from theoretical models to empirical observations, Hoffmann, Shefrin, Pennings in 2010 state that the importance of considering the latent heterogeneity amongst investors their preferences and beliefs should form the underlying drivers of their behavior. Being aware that psychological processes drive investment choices, they indicate limits of indirectly deducing them through observable socio-demographic variables. Nevertheless, authors mainly 'infer' psychological traits from investment style. Precisely, they assume that investment objectives should reflect investor preferences, and that investment strategy should reflect investor beliefs. Even if they indicate a relevant step ahead towards the empirical testing of Behavioral Portfolio Theories, these assumptions bring them to implicitly neglect the 'latent heterogeneity' that motivated their study. This is mainly due to a lack of experimental measures that are able to describe the specific psychological process that impacts upon the manner in which individuals make investment choices.

Recent technological developments have increased the set of disciplines interested in exploring human economic decision-making, including neuroscience (Rustichini (2005)). Neurological experiments have shown evidence of the role of emotions while taking risky choices (Loewenstein (2000), Damasio (1994)). Innovative technological advancements have led to a 'quantum jump' in the knowledge of the physiology of human decision making. For example Preuschoff, Quartz, Bossaerts, (2008) offer brain-scanning (fMRI) evidence that activity in certain sub-cortical structures of the human brain correlate with changes in expected reward, as well as with risk, which is measured by variance of payoff, as is described in Markowitz theory. Their findings suggest that the brain may perform a higher dimensional analysis of risky gambles, and that the human brain appears to record inputs of the MVT.

Further innovations integrate fMRI images with psycho-physiological measures, in particular Skin Conductance (Wong, Xue and Bechara (2011)). These authors suggest that psycho-physiological data would complement fMRI findings in providing a more comprehensive understanding about the physiological and neural mechanisms of decision making.

This paper adds to the existing literature in that it exploits both intuitions of Behavioral Portfolio Theories and measurements of heterogeneous emotional activations. It offers a theoretical generalization of a relevant economic theory and validates its accuracy, in comparison to the classical one. The paper concludes that there is no antagonism between micro-economic theories of rational choice and scientific evidence of emotional behaviors, because by merging them we are able to describe/predict the real human optimizing decision process.

3 Theoretical Models

The 'expected returns-variance of returns' rule of the MVT is based on a simple assumption: expected return is 'a desirable thing and variance of return an undesirable thing' (Markowitz, 1952 p. 77). Nevertheless, this model considers expected returns and variance of return as the *unique* desirable and undesirable thing, respectively. Conversely, we add that decision making is driven by a further variable, regarding not only monetary pay-off, but also emotional reactions to choices.

Coherently, we define two models: firstly, a 'normative' model, based on the Markowitz approach, which follows the assumption that individuals' portfolio choices are uniquely driven by the 'expected returns-variance of returns' rule; secondly, a 'generalized' model, which allows us to include emotions within the individuals' decision process. In this model we refer to a general measure (e) of emotional activation that individuals experience when making risky choices.

Let us now define the further assumptions of these models. Let us assume that n assets are available with random return distributions. The following variables are contained in the model:

- x : the column vector of portfolio weights.
- r_{ij} : the column vector of returns of asset i, ($i = 1, \ldots, n$), for the j^{th} agent.
- μ_j : the column vector containing the means of the returns r_{ij} of the n assets for the j^{th} agent.
- j : the i.d. of the single agent.
- e_j : the column vector of emotional responses of the j^{th} agent after each choice.
- $r(e_{ij})$: the column vector of returns of asset i for the j^{th} agent with. The interpretation of $r(e_{ij})$ is immediate: these returns represent the 'subjective' reward of individuals when considering both the monetary

returns and the emotional compensation. In this case, returns are a certain function f of the individual's emotional response e_j. The choice of the function f will be discussed in the empirical section. Note that, the choice of f is independent from the agent j, i.e. the agent is totally defined by his/her specific e.

- μ_{e_j} : the column vector containing the means of the returns $r(e_{ij})$ of the n assets for the j^{th} agent.
- D_j : the covariance matrix of returns r_{ij}.
- D_{e_j} : the covariance matrix of returns $r(e_{ij})$.

Finally, we assume that agents are not able to observe simultaneously the performances of n assets: when choosing the n^{th}, all the other $n-1$ are neglected.

The optimization problem for the j^{th} agent, within the 'normative' model, is the following:

$$\begin{aligned} &\min_x \quad x'D_jx \\ s.t. \quad &x'\mu_j = \mu_P \\ &x'\mathbf{1} = 1 \end{aligned} \tag{1}$$

where, μ_P is a given level of portfolio return and $\mathbf{1}$ is a column vector of ones.

The optimization problem for the j^{th} agent, within the 'generalized' model, is the following:

$$\begin{aligned} \min_{x} \quad & x' D_{e_j} x && (2) \\ s.t. \quad & x' \mu_{e_j} = \mu_P \\ & x' \mathbf{1} = 1 \end{aligned}$$

This second model is a generalization of the first one, because it includes e_j through the function f. This represents an additional parameter in the optimization process. Note that the two models coincide when:

- the function f is constant, i.e. the individual parameter e_j does not play any significant role in the model.
- e_j is constant. The agent does not show any significant emotion when facing the choice of risky investment[2].

No short positions are allowed, in order to shape a theoretical contest that is coherent with the following empirical validation. Therefore, both models are developed with the following restriction:

$$0 \leq x_i \leq 1, \qquad for \quad i = 1, \ldots, n.$$

[2]In this case the choice of the function f plays the role of a numeraire, to induce a sort of 'scale' effect. The results of the two models are numerically different but substantially equal, as shown in the empirical section

4 The Experiment: Methods and Sample

During model validation, sampled individuals were asked to build their own portfolios through a series of choices which we assume are driven by asset risk/return information, in the 'normative', model (1), and by both the asset risk/return information and their emotional experience, in the 'generalized', model (2). Description of our experiment is offered in the Appendix. For each agent, we compare portfolio choices with efficient frontiers obtained from the two models. We consider an 'efficient' portfolio, for an individual, any combination of assets that lies along the efficient frontier.

Our methodology strictly replicates in a laboratory setting the two stages that Markowitz states guide individual portfolio choices. We set a learning/training period (first set of choices) that allows individuals to learn the risk/rewards dynamics of the experiment. It is reminiscent of the 'first stage' of the MVT, when an individual 'starts with observation and experience, and ends with beliefs about the future performances of available securities.' (Markowitz, 1952, p.77.) Coherently, we set a testing period (last set of choices) that corresponds with the MVT 'second stage' which 'starts with the relevant beliefs about future performances and ends with the choice of portfolio' (Markowitz, 1952, p.77).

The experiment involved an assorted sample of individuals: customers of banks and financial professionals (traders, asset managers and financial advisors). More than 900 individuals were asked to take part in our experiments and 645 of them did so, with neither obligation nor reward. We checked for self-selection biases. The width of the sample is relevant, considering the use of psycho-physiological tests, such as the measurement of SCR. For example, in similar experiments, Lo and Repin (2002) examined 10 subjects; Lo, Repin and Steenbarger (2005) studied 33 individuals; Bechara and Damasio (2002) compared 46 substance dependent individuals, 10 subjects with lesions of the ventromedial prefrontal cortex and 49 normal controls.

Each individual is asked to take 100 choices and the task duration is about forty-five minutes for each participant (see the Appendix).

The laboratory setting allows us to control for the personal knowledge or experience of individuals towards specific financial assets. Any difference in this background may influence portfolio choices, and 'disturb' the role of the crude 'expected returns-variance of returns' rule. Coherently, individuals were asked to collect a portfolio by selecting from a range of 4 anonymous assets: we do not refer to a typology of financial asset, such as bond or stock, but propose generic 'A', 'B', 'C' and 'D' assets, each with a different random risk/return combination. Pay-offs of our experiment refer to traditional monetary return and risk variables.

During the experiment we measure the intensity of emotional reactions by using the SCR, which we take as a proxy for e. We are not interested in distinguishing the 'nature' of emotions, in terms of positive (pleasure) or negative (pain) experiences; we uniquely consider their 'intensity', as those 'visceral factors' (Lowenstein (1996)) involved in decision making.

Precisely, SCR measures the voltage drop between two electrodes placed on the skin surface of the subject during the experiment (Figner and Murphy 2010), as shown in Figure 4 of Annex. Changes in SCR occur when the eccrine sweat glands, which are innervated by the sympathetic autonomic nervous system fibers, receive a signal from a certain part of the brain. Recording of SCR starts at least ten minutes before the beginning of the experiment and continues throughout. The sample rate is set at 1 Hz.

5 Validation of Models

We assume that individuals are sensitive to expected payoff and risk, represented by historical expected return and historical variance, as in model (1); or alternatively by a combination of these variables with emotional activation, as in model (2). This means that the agent's utility function depends exclusively from the first two moments of returns' distribution.

Given the length of the empirical task, i.e. a sequence of 100 selections, we set the first 80 choices as the learning period lp, and the last 20 choices, as the testing period tp. A 70-30 cut-off has been considered, as well, as a robustness check.

The experience of choices and pay-offs is unique, for each agent, and describes a specific pattern of selections that drives towards individual efficient frontiers, in the learning period, and towards individual portfolios, in the testing period. We draw an efficient frontier in the mean-variance space, for each agent, on the basis of her first 80 choices. The last 20 choices indicate the frequency of the 4 assets and allow us to obtain the portfolio which is definitely selected by each individual, after the training experience. Therefore, our efficient frontiers materialize as solutions of the optimization problem that alternatively neglects, in model (1), or includes emotions, in model (2), as far as the individual learning process is concerned. Conversely, testing portfolios are results of this learning process and do not depend upon models.

It is apparent that the validation of our models is obtained individually: for each agent, we observe the 'specific' portfolio's positioning compared to her 'specific' efficient frontier. This is done for both the 'normative' and the 'generalized' model. If the testing portfolio belongs to the efficient frontier, the agent is considered to be efficient, independently from the portfolio's positioning on the frontier itself, given that no assumptions are made on agents' risk aversion. Conversely, if the testing portfolio does not lie on the efficient frontier, the agent is classified as sub-efficient.

As testing portfolios do not depend on models and are 'fixed' in terms of composition by each agent, sub-efficiency can only be deduced when a referring model has been set, i.e. we have drawn the efficient frontiers, by using model (1) or model (2), alternatively.

The lack of efficiency in the agent's portfolio can also be interpreted in terms of limited accuracy of the model itself: if an agent is sub-efficient it is equivalent to claim that the model is not able to describe the individual decision process.

5.1 The 'Normative' Model: Validation for a Random Sample of Individuals

First we validate the 'normative' model (1) by comparing individuals' efficient frontiers and testing portfolios in the mean-variance space, within a 80-20 cut-off. Given that validation is driven individually, we obtain 645 efficient frontiers, with their specific testing portfolios. Any information about efficient or sub-efficient positioning of individuals is obtained by checking if individual testing portfolios lie on their 'specific' efficient frontiers.

Given the number of agents we analyzed, in this section we offer a limited view of our results, with reference to a random sub-sample of individuals: we select the first four subjects that took part in our experiments.

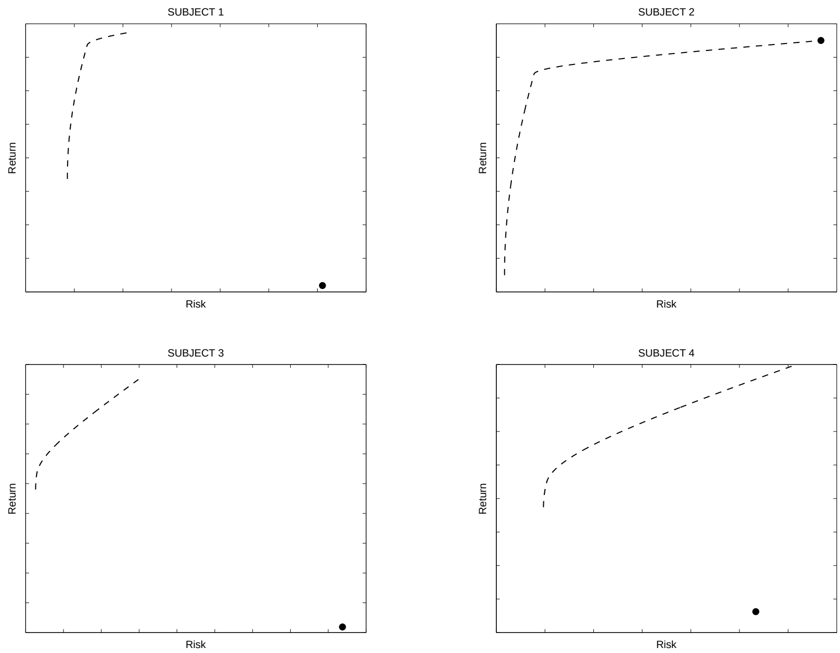
The first piece of evidence, depicted in Figure 1, is that 3 out of 4 testing portfolios are sub-efficient, in relation to efficient frontiers obtained from the 'normative' model. Possible explanations of innacuracy of the MVT model include:

a) individuals do not learn from historical expected returns and variances, i.e. this information is not sufficient to guide their choices efficiently;

b) individuals do learn from historical expected returns and variances, but they are not willing to optimize, i.e. they neglect to minimize variance of return for any given expected return;

Figure 1: Validation of 'normative' model: random sub-sample

Efficient frontiers and testing portfolios are drawn individually, so that each chart refers to a specific subject (Subject 1 to Subject 4). We draw an efficient frontier, for each agent, with a dotted line, on the basis of his or her first 80 choices. Here, frontiers result as solutions of the optimization problem of model (1), that neglects emotions. The last 20 choices allow us to obtain the portfolio that is selected by each individual, indicated with the • in the mean-variance space. The 'Y-axis', namely Return, indicates, historical expected returns; the 'X-axis', namely Risk, indicates their historical variance. We omit to indicate the unit scale because it varies by individual, even if this does not affect results, which arise from observation of the • positioning respect to its specific frontier.



c) individuals do learn from historical expected returns and variances, plus they would like to optimize, but they are incapable of doing so, i.e. they lack the technical capabilities that would enable the calculation of efficient portfolios.

From this standpoint, we have reasons to doubt that MVT reflects how investors practically behave, because individuals may not be able to learn from historical expected returns and variances (sub a), or not willing (sub b), or not able (sub c), to take decisions that follow an optimization process.

Nevertheless, before reaching the conclusion that the Markovitz conceptual framework fails to describe reality, we need to validate the 'generalized' model.

5.2 The Choice of Function f

The validation of the 'generalized' model (2) must be preceded by a comment on function f. This function merges returns with individual emotions e, here proxied by SCR. Function f transforms 'objective' returns into 'subjective' ones. The form of function f has been designed according to the prospect theory proposition of Kahneman and Tversky (1979) on the value function, which is 'generally concave for gains and commonly convex for losses '(p.279). We approximate the shape of their hypothetical value function with a cubic function. Coherently, we weight the monetary ('objective') returns with the cubic of the emotional response and obtain the 'subjective' returns as follows:

$$r_t(e_j) = \frac{r_{tj}}{(e_{tj})^3} \tag{3}$$

where t represents the t^{th} entry of the vector $r(e_j)$ and e_{tj} represents the activation of the j^{th} agent after the t^{th} choice.

One might claim that our results are just induced by the choice of function f. This argumentation is false because of the heterogeneous behaviors of individuals: each individual experiences her own pattern of choices, and consequently of both returns and emotional activations. Function f could have induced specific results for *one* individual. On the contrary, *our* function f is indifferently applied to 645 individuals and it is effective for *all of them.*

5.3 The 'Generalized' Model: validation for a random sample of individuals

Validation of the 'generalized' model is performed like before, however with the difference that any specific efficient frontier is obtained from model (2), i.e. individuals learn by both asset risk/return information and individual emotional experience. As for the 'normative' model, here efficient frontiers are obtained from the first 80 choices of the learning period, lp; and testing portfolios result from the frequency of choices taken during the last 20 choices (tp), among the 4 alternative assets.

It is important to underline that emotional measures intervene exclusively in solution of the optimization problem of model (2), that results in drawing efficient frontiers from the first 80 training choices. After this set of choices, the learning process is deemed to be finished. Therefore, testing portfolios uniquely emanate from the frequency of assets, during the last 20 choices.

In Figure 2 we show efficient frontiers and testing portfolios in the mean-variance space, for the same sub-sample of four individuals of Figure 1.

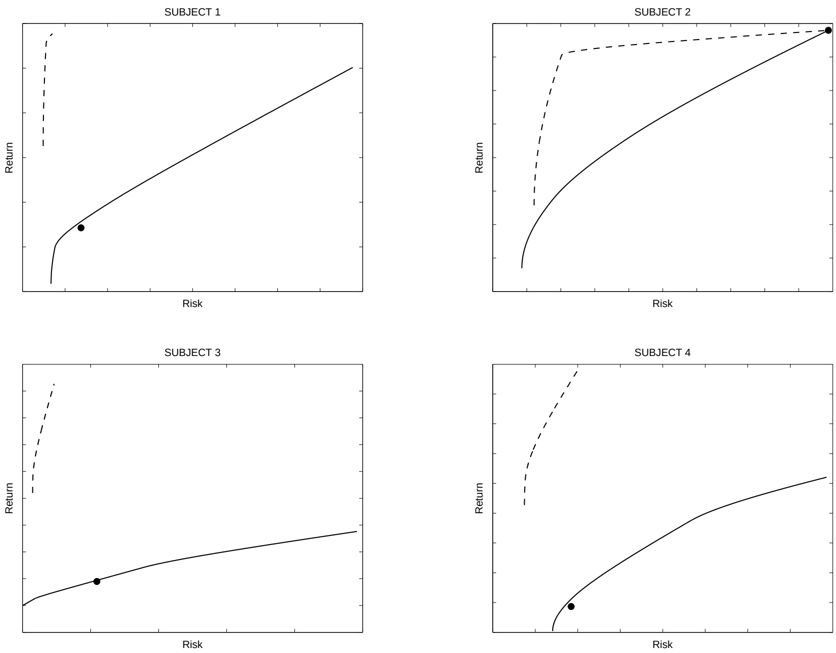
Results appear markedly different: testing portfolios of all our four individuals can be considered efficient, from the graphical interpretation, as their position in the mean-variance space is on the efficient frontier, or very close to it[3]. Subject 2 is efficient for both models; Subjects 1, 3, 4, move from sub-efficiency in the 'normative' model, to a position of efficiency in the 'generalized' model.

These findings allow us to suppose that the Markovitz conceptual framework fails to describe reality only when it is limited to return/risk information, while it is able to describe investors' behavior more accurately when it is generalized and when it includes both return/risk and emotional information. In order to support this argument, we should also examine the numerical evidence relating to the entire sample.

[3]Sometimes testing portfolios do not fall precisely onto frontiers because of discreteness of weights (frequency) of assets that could be selected (20 choices). We will discuss this problem of granularity in Section 7.

Figure 2: Validation of the 'generalized' model: random sub-sample

For our four subjects, we draw both the efficient frontier (dotted line) originating from the solution of the optimization problem of model (1) that neglects emotions, and the efficient frontier (continuous line) originating from the solution of model (2) that includes emotions. Testing portfolios, indicated with the • , are fixed in the mean-variance space, as frequency of choices (weights of assets) during the last 20 choices do not depend from the optimization model. The 'Y-axis', namely Return, indicates, historical expected returns; the 'X-axis', namely Risk, indicates their historical variance. Again, as in Figure 1, we omit to indicate the unit scale because it varies, by individuals, and it is affected by function f; nevertheless, this does not affect results, which arise from observation of the • positioning respect to the two efficient frontiers.



6 'Normative' versus 'Generalized' Model: Overall Relative Efficiency

Observations of individual testing portfolios compared to specific efficient frontiers of each agent in the mean-variance space, are striking when considering the small amount of individuals used in the sub-sample of Figure 2. In order to extend results to include all 645 individuals, we make use of the relative portfolio efficiency measure introduced by Kandel and Stambaugh (1995). This measure is used to quantify distances of portfolios from efficient frontiers[4]. The ϕ of Kandel and Stambaugh (1995) is:

$$\phi_j = \frac{\mu_j - \mu_g}{\mu_x - \mu_g} \tag{4}$$

where j stands for the j^{th} agent, μ_j is the expected return of the testing portfolio, μ_g is the expected return of the minimum variance portfolio and μ_x is the expected return of the efficient portfolio with the same risk of the testing portfolio. The value of ϕ_j belongs by construction to the interval $[-\infty, 1]$. If $\phi_j = 1$ the individual portfolio belongs to the efficient frontier, $\phi_j = -1$ the individual portfolio belongs to the inefficient part of the frontier, while higher negative values of the index represent 'severe' sub-efficiency.

Table 1 shows the deciles of the empirical distribution of function ϕ for the two models. The left series of deciles refers to the 80-20 cut-off, as a

[4]The introduction of individual emotional activation produces large differences of scale from one model to another, and from one individual to another; this measure of relative portfolio efficiency permits direct comparability of results.

training sequence of choices; the right series of deciles refers to the 70-30 cut-off. In general, the best fit of the models is obtained when the distribution of ϕ collapses on the value 1.

Table 1: Deciles of the distribution of ϕ.

This Table shows the empirical distribution of the ϕ of Kandel and Stambaugh (1995), when computed under the model 1, namely (ϕ), and model 2, namely ($\phi(e)$). Left series of deciles refers to the 80-20 cut-off, for the training and testing period; the right series of deciles refers to the 70-30 cut-off.

80-20 Training-Testing Cut-Off			*70-30 Training-Testing Cut-Off*		
Deciles	Model 1 ϕ	Model 2 $\phi(e)$	Deciles	Model 1 ϕ	Model 2 $\phi(e)$
0	-57789.19	-449.04	0	-1.53E+17	-1325.8
10	-16.83	-0.91	10	-19.87	-1.03
20	-9.8	0.39	20	-11.92	0.3
30	-7.08	0.67	30	-7.82	0.71
40	-5.11	0.81	40	-5.56	0.81
50	-3.7	0.86	50	-3.94	0.88
60	-2.66	0.92	60	-2.66	0.91
70	-1.78	0.95	70	-1.69	0.94
80	-1.04	0.98	80	-0.92	0.97
90	0.21	1	90	-0.02	0.99
100	1	1	100	1	1

Results are robust for different cut-off periods. Values from 'normative' models show that the 90^{th} deciles of the distribution correspond to large sub-efficiency, with a ϕ of 0.21 for the 80-20 cut-off, and a ϕ of -0.02 for the 70-30 cut-off. Conversely, in the 30^{th} deciles of the distribution for the 'generalized' models, ϕ_e values are close to efficient levels, with a ϕ_e of 0.67 for the 80-20 cut-off, and a ϕ_e of 0.71 for the 70-30 cut-off. From this perspective, , this means that approximately only 10 per cent of individuals take efficient decisions within the MTV framework, while at least 70 per cent of individuals take efficient decisions when the MTV framework is integrated with the emotional component.

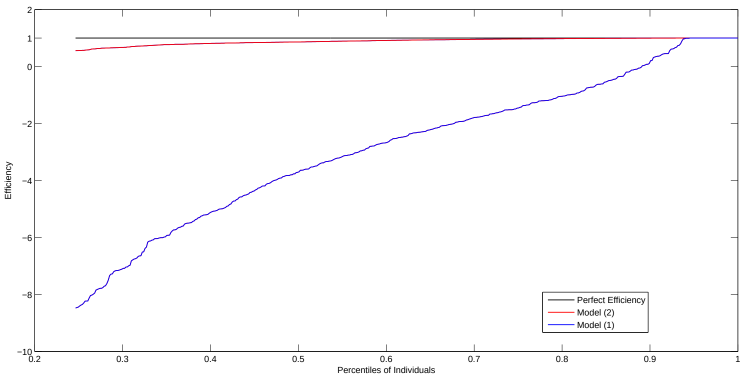
The condition of 'severe' sub-efficiency, that is when ϕ is lower than -1, marks further strong differences between the two models: this condition affects 507 individual portfolios (79 per cent of the sample) and 519 individual portfolios (80 per cent), within the Markowitz 'normative' model, for the 70-30 and 80-20 cut-off, respectively. Conversely, within the 'generalized' model, portfolios suffer 'severe' sub-efficiency for only 65 individuals (10 per cent) and 61 individuals (9 per cent), for the same two cut-off periods.

Figure 3 reinforces these results as it shows values of ϕ (blu line) and ϕ_e (red line) by deciles, and their different progressive trend towards 'level 1', i.e. perfect efficiency. Given the asymmetry of the theoretical values of function ϕ, we discard the lowest deciles because they correspond to negative values which are totally out of scale. Figure 3 compares results by different cut-off periods and confirms the robustness of our findings: the ϕ_e distribution (red line) appears immediately (from the lowest deciles) lying on 'level 1', which indicates efficiency, while the ϕ distribution (blu line) reaches 'level 1' only for the 90^{th} decile.

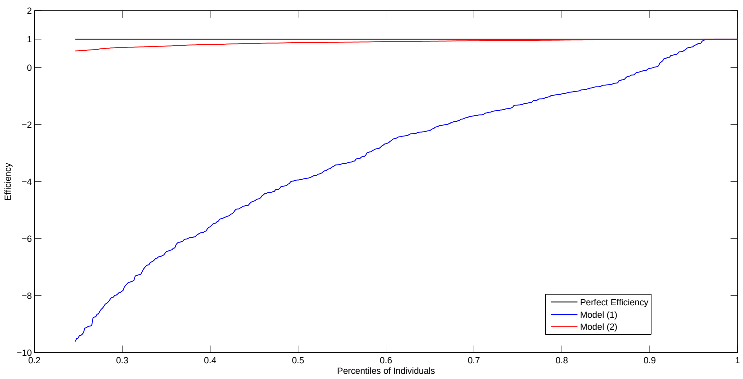
Figure 3: Trend of ϕ and ϕ_e by deciles

We sort individuals by increasing values of ϕ and ϕ_e. We exclude from charting the first 20 per cent of individuals with negative values out of scale. The blue line indicates the ϕ trend by percentiles of individuals; the red line indicates the same trend for ϕ_e. We show results for both 80-20 and 70-30 training-testing cut-off periods.

80-20 Training-Testing Cut-Off



70-30 Training-Testing Cut-Off



7 Efficiency and Granularity of Portfolio Choices

The drawing of efficient frontiers in the mean-variance space requires the assumption of infinite divisibility of assets. This is consequential to the optimization process that produces investments' weights of efficient portfolios in terms of continuous numbers. When moving from theory to actual-investing, in the real world, efficient portfolios are frequently not feasible, because they would require the splitting of assets into weights that would not be practicable.

In the validation of the models, constraints to infinite divisibility of assets are set by the number of possible choices c during the learning and testing periods. For example, a testing period tp of 20 choices implies that the minimum share for each asset is equal to 1/20 ($1/c_{tp}$).

Granularity of testing portfolios can reasonably affect their efficiency in terms of ϕ. For this reason, we introduce a condition for ϕ, in order to check, for each agent j, if the corresponding ϕ_j is significantly different from 1. This condition allows us to distinguish testing portfolios that are 'discrete' approximations of efficient ones, from true sub-efficient portfolios. The first group refers to portfolios that are not significantly different from efficient portfolios; the second group includes portfolios that are significantly sub-efficient.

Condition for ϕ_j is obtained by adding an incremental component to the testing portfolio induced by the granularity of testing choices. Precisely, given x_j the testing portfolio for the j^{th} agent, and c_{tp} the number of testing choices, x_j is not considered significantly different from an efficient portfolio if a portfolio x^* exists such that:

$$\phi(x_j + x^*) = 1$$
$$s.t. \qquad (x^*)'\mathbf{1} = 0$$
$$|x_i^*| < \frac{1}{c_{tp}} \quad with \quad i = 1, \dots, n$$

We verify this condition under both validated models. In Table 2 we presents values of ϕ_j and $\phi(e)$, for the 'normative' and the 'descriptive' model, respectively, with reference to the sub-sample of the first 75 individuals taking part in our experiment. We mark with * those individuals whose testing portfolio cannot be considered significantly different from efficient ones (values of ϕ are not significantly different from 1). It is immediately observable that the first four agents correspond to those depicted in Figure 1 and Figure 2.

We now have a condition to state the overall level of accuracy of model (1) compared to model (2): as far as the 80-20 cut-off is concerned, under model (1) only 13 per cent of testing portfolios can be considered efficient, when under model (2) 85 per cent of them can be considered 'discrete' approximations of efficient portfolios. These percentages are consistent within the 70-30 cut-off, where only 11 per cent of testing portfolios are efficient for model (1), against 84 per cent of portfolios that can be considered efficient under model (2).

Table 2: Comparison of the values of ϕ by individuals and models.
*This table shows individual values for ϕ, related to model (1), and for $\phi(e)$, related to model (2). We specify with the * mark whether the value can be considered not significantly different from 1, i.e., the portfolio is a 'discrete' approximation of an efficient portfolio. We show a selection for the first set of 75 subjects.*

ID	ϕ	$\phi(e)$	**ID**	ϕ	$\phi(e)$	**ID**	ϕ	$\phi(e)$
1	-0.73	0.90*	26	-1.52	0.97*	51	-3.55	0.10*
2	1.00*	1.00*	27	0.99*	0.98*	52	-7.80	0.70*
3	-1.25	0.95*	28	-0.45	0.77*	53	-7.78	0.54*
4	-0.81	0.77*	29	-37.24	0.98*	54	-23.42	1.00*
5	0.21*	0.91*	30	-7.11	-3.71	55	-5.10	-1.72
6	0.31*	-0.82	31	-0.09*	0.84*	56	-11.88	0.83*
7	-8.64	0.89*	32	-2.20	0.89*	57	-2.01	0.86*
8	-1.38	0.17*	33	-15.09	0.96*	58	1.00*	0.85*
9	-1.83	-0.91	34	-5.60	-0.55	59	-5.15	0.57*
10	-2.70	0.94*	35	-3.80	0.88*	60	-6.01	0.36*
11	-4.52	0.88*	36	-1.62	0.98*	61	-4.99	0.93*
12	-6.04	-2.61	37	-51.75	-2.50	62	-8.70	0.98*
13	-3.42	0.91*	38	1.00*	1.00*	63	-8.60	0.89*
14	-9.21	0.89*	39	-2.07	0.85*	64	-6.82	0.98*
15	-13.60	0.14*	40	-5.12	0.85*	65	-4.99	-6.12
16	0.42*	0.65*	41	-33.36	1.00*	66	-3.02	0.71*
17	0.66*	0.94*	42	-30.88	-69.23	67	-3.15	0.86*
18	-16.70	-0.29	43	-3.38	0.88*	68	0.62*	0.98*
19	-3.21	0.86*	44	-3.11	0.96*	69	1.00*	0.83*
20	-1.52	0.43*	45	-5.51	0.95*	70	-15.27	0.78*
21	-1.52	0.92*	46	-0.42	0.99*	71	-67.11	1.00*
22	-1.46	0.82*	47	-7.62	0.98*	72	-66.87	-449.04
23	-14.40	-0.78	48	1.00*	1.00*	73	-7.13	0.97*
24	-1.76	0.86*	49	1.00*	0.84*	74	0.97*	-4.86
25	-1.90	0.90*	50	-6.12	0.66*	75	1.00*	0.93*

8 Discussion and Conclusions

This paper offers a theoretical generalization of the MVT decision making framework by integrating the 'expected return-variance of returns' rule with emotions. The accuracy of the 'generalized' portfolio model is compared to the classical model. The evidence shows that the Markowitz model frequently fails to describe 'what an investor practically does', most likely due to the fact that individuals are not able to learn from historical expected returns and variances, or that they seldom follow an optimization process - when only monetary pay-offs are considered. They might also lack the technical capabilities to calculate efficient portfolios.

Nevertheless, descriptive limits of the original Markowitz model do not imply that its framework for human decision making does not hold.

Unambiguous evidence from the validation of our 'generalized' model proves that individuals actually follow an optimizing decision process and take efficient portfolio choices, but only when emotions are added to the equation. It follows that the human optimization process is not limited to monetary gains and losses, but it is also driven by additional emotional gratification. In this regard the MVT conceptual framework properly describes efficient behaviors, but its complete formulation should incorporate visceral components.

The descriptive power of the 'generalized' model appears so strong in terms of both a large presence of efficient portfolios and scarcity of 'severe' sub-efficiency, that we also attest to its accuracy in being able to forecast individual choices. Our findings support the notion that if we repeated our experiments and trained individuals with an initial 80 (or 70) frequency of choices, with the crucial support of the emotional activation we would be able to forecast, with a 85 (or 84) per cent of confidence, the specific frontier in which their further 20 (or 30) choices will fall. We are not able to forecast the precise coordinates of these portfolios in the mean-variance space, but we can foresee their specific orbit, in this space.

A suggestive interpretation of the overall findings of this paper advocates that emotional activation not only leads to choices, as already suggested by the Somatic Marker Hypothesis of Damasio (1994), but it leads to 'efficient' choices.

The key point of deliberation is: which kind of 'efficiency' are we dealing with? Firstly, it is an 'economic' efficiency, as it results from an optimization process. Secondly, it is a 'subjective' efficiency, because each individual has her own unique level. Thirdly, it is a 'relative' efficiency, because it results from comparing the position of testing portfolios with 'different' efficient frontiers, that change in relation to different optimization models.

In summation, this paper proposes a new 'Somatic Portfolio Theory', where emotions and body' signals lead to efficiency in decisions of economic value. Its sober theoretical layout allows developments in its formalization, as well as further studies that could test alternative measures for emotional activation, with respect to SCR, or that could fine tune the f function. Our model effectively eliminates much of the antagonism between micro-economic theories of rational choice and scientific evidence of emotional behavior, because it shows that by merging them, the authentic human optimizing decision process can be both described and predicted.

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Appendix: Description of the Experiment

Each individual is asked to select a card among four decks, which represent our investment opportunities. We wished to avoid any framing effect due to personal knowledge or experience of individuals towards specific financial assets. We do not refer to a typology of financial asset, such as bond or stock, but propose generic 'A', 'B', 'C' and 'D' assets (i.e. our four decks), each with a different risk/return combination.

Before the task, participants are not given information about how many choices they are supposed to make; they can change deck whenever they wish. The goal of the task is to gain as much money as possible and to avoid losing money as far as possible.

In order to perform the task, subjects are given some short verbal instructions, written on the computer screen when they seat in order to run the experiment:

'You see in your screen four decks of cards: A, B, C, and D. I want you to select one card at a time, by clicking on it, from any deck you choose. After each card selection, the computer will tell you that you won or lost some money. You are absolutely free to switch from one deck to another any time you wish. The goal of the game is to gain as much money as possible and, if you find yourself unable to gain, make sure you avoid losing money as much as possible. I won't tell you for how long the game will continue. You must keep on playing until the computer stops.'

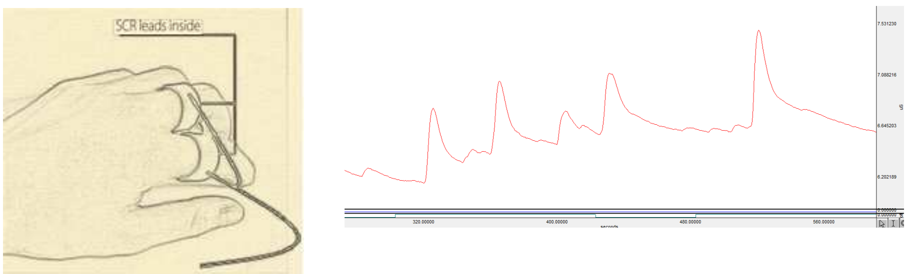
Effectively, participants make a sequence of 100 choices, and receive a monetary outcome after each selection, in terms of game money.

Table 3: Moments of the payoff distribution of the four decks-investments.

	A	B	C	D
Expected payoffs	-28.233	-31.9334	26.44745	28.44987
Standard deviation of payoffs	136.6131	384.0835	26.86451	70.16822

Figure 4: The Skin Conductance Response measurement

The left figure shows the two electrodes placed on the skin surface of the agent running the experiment. Electrodes are attached to the palm surface of the second phalanx of the index and middle fingers of the non-dominant hand, after the subject is seated in a comfortable chair in front of the computer screen. The right chart shows the typical trend of SCR during the experiment: the upward trends of SCR correspond to the somatic reactions of individuals to choices; the downward trends correspond to the recovery of SCR towards the individual's baseline, when the computer stops, from a choice to another.



The pay-offs from the four decks appear to be a good simplification in order to investigate individual choice processes in mean-variance framework. Decks A and B are strictly dominated in terms of mean-variance criterion by decks C and D. Moreover, B is strictly dominated by C. On the other hand, there is no trivial ordering between C and D, because the higher risk for D is counterbalanced by its higher expected pay-off.

The portfolio which is composed by the sequence of selections is specific for each individual, because it results from the precise pattern of preferences that she takes, during the experiment. In the extreme case a subject selected from a unique deck, the risk-return profile of this portfolio would be that shown in Table 3.

The Skin Conductance Response is measured by the voltage drop between two electrodes placed on the skin surface of the agent running the experiment, as shown in Figure 4. Changes in SCR occur when the eccrine sweat glands, which are innervated by the sympathetic autonomic nervous system fibers, receive a signal from a certain part of the brain. Recording of SCR starts at least ten minutes before the beginning of the task, and continues throughout. The computer tracks the sequence of the cards selected from the various decks. Each time the subject clicks the mouse to select a card during that time interval, the computer will not respond, and therefore no record is generated. As the subject performs the task, SCR activity is recorded continuously and collected simultaneously on another personal computer, where data of the experiments are stored. Sample rate is set at 1 Hz. Each time the subject selects a card, this action is recorded as a 'mark' on the polygram of SCR activity. Each click is registered as a selection from the specific deck that was chosen. Thus, SCR generated in association with a specific card, from a specific deck, can be identified precisely on the polygram. The intertrial interval is set at six seconds, and a 'break' phase is included in order to allow the SCR to decrease and recover towards the normal individual baseline. Nevertheless, in order to allow for psycho-physiological recordings, the time interval between two card selections is longer, because it takes a few additional seconds for the subject to decide which card to pick next. This time interval varies from trial to trial. It is on average ten seconds. The overall task duration varies from about thirty to forty-five minutes, in relation to the specific speed reaction of each individual.

Note that each individual owns a specific SCR baseline, which is very different from one subject to another. Our measure e does not refer to an absolute level of SCR, which might be related to this biological difference, but it is obtained as a relative measure, related to the intensity of the SCR variation, due to the choice, from her individual specific baseline.